Sparsity-Enhanced Convolutional Decomposition: A Novel Tensor-Based Paradigm for Blind Hyperspectral Unmixing

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Abstract—Blind hyperspectral unmixing (HU) has long been recognized as a crucial component in analyzing the hyperspectral imagery (HSI) collected by airborne and spaceborne sensors. Due to the highly ill-posed problems of such a blind source separation scheme and the effects of spectral variability in hyperspectral imaging, the ability to accurately and effectively unmixing the complex HSI still remains limited. To this end, this article presents a novel blind HU model, called sparsity-enhanced convolutional decomposition (SeCoDe), by jointly learning spatial-spectral information of HSI in a tensor-based fashion. SeCoDe benefits from two perspectives. On the one hand, the convolutional operation is employed in SeCoDe to locally model the spatial relation between the targeted pixel and its neighbors, which can be well explained by spectral bundles that are capable of addressing spectral variabilities effectively. It maintains, on the other hand, physically continuous spectral components by decomposing the HSI along with the spectral domain. With sparsity-enhanced regularization, an alternative optimization strategy with alternating direction method of multipliers (ADMM)-based optimization algorithm is devised for efficient model inference. Extensive experiments conducted on three different data sets demonstrate the superiority of the proposed SeCoDe compared to previous state-of-the-art methods. We will also release the code at https://gitub.com/danfenghong/IEEE_TGRS_SeCoDe to encourage the reproduction of the given results.

Index Terms—Blind hyperspectral unmixing (HU), convolutional sparse coding (CSC), spectral bundles, spectral variability (SV), tensor decomposition.

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I. INTRODUCTION

A IRBORNE or spaceborne imaging spectroscopy, also known as hyperspectral imaging, plays a fundamental role in earth observation and remote sensing. Characterized by abundant spectral information [1], hyperspectral imagery (HSI) has been favored by researches upon various applications, such as dimensionality reduction [2]–[4], land-cover and land-use classification, object detection [5]–[10], spectral unmixing [11]–[13], image segmentation [14], and multisource data fusion [15]–[17]. Nevertheless, due to the limitations of spatial resolution in hyperspectral imaging, the pixels in HSIs are usually in the form of mixtures, inevitably degrading the performance of the subsequent high-level analysis. Such a challenge motivates the constant development of spectral unmixing, which aims at decomposing the mixed spectrum into a collection of spectral signatures of pure materials (or say, endmembers) and their fractional abundances (or say, abundance maps).

A wealth of hyperspectral unmixing (HU) methods has emerged apace with the advances in statistically modeling and machine learning within the past few decades. The focus of this context starts from a relatively simplistic yet practical family of the model, i.e., the linear mixing model (LMM) [18]. Assuming that microscopic interaction between distinct materials is negligible, LMM basically seeks a linear decomposition to approximate the mixing process of the collected hyperspectral data. This kind of HU method largely depends on the endmember extraction algorithms. Except for the popular vertex component analysis (VCA) [19], Zhang et al. [20] for the first time proposed to use the ant colony optimization (ACO) to intelligently extract the endmembers of HSIs, which has been recently recognized as a pioneering work in the HU field and achieved the state-of-the-art unmixing results in many real hyperspectral scenes. The majority of recently proposed LMM-based methods rely on nonnegative matrix factorization (NMF) due to its good interpretability and efficient solution [21]. Besides, several kinds of regularization terms that are capable of encoding certain prior knowledge have been successfully employed from structure sparsity to spatial smoothness [22], [23]. Despite so, the effectiveness of these linearized blind HU methods remains limited, particularly in complex real scenarios where there exist various spectral variabilities.
The spectral variability (SV) refers to the phenomenon that the spectral reflectance of a given endmember can vary spatially or spectrally, due to multiple intrinsic and exterior factors (e.g., atmospheric effects, illumination conditions, geometry, and topography) [24]. To eliminate the negative effects on HU performance caused by such unpredictable variations, numerous efforts have been made to develop either stochastic or deterministic models (see [25], [26] for a recent review). Still, in the case of LMM, a representative category of studies resorts to spectral bundles, by representing each material within a known or estimated set of endmember “candidates,” to account for the variability [27]–[29]. More recently, physics-inspired models that extend LMM in a more constrained way to explicitly characterize SVs have received considerable attention [30], [31]. Albeit verified to be effective to some extent, these methods potentially suffer from the mismodeling effects by their vector-/matrix-based implementations.

Considering the highly ill-posed problem of blind HU, the loss of spatial correlation information brought by roughly stacking the spectrum of each mixed pixel is not tolerable. Fortunately, tensorized formulation has shown great potential in providing a faithful representation for multidimensional data. Recently proposed tensor factorization-based methods for blind HU task, by modeling the observed HSI data cube as a third-order tensor, are capable of preserving the inherent spatial–spectral information and achieving a satisfying performance [32], [33]. However, two issues are worthy of our consideration. The first issue brings failure to characterize the SV. Second, there is still room in making full use of structural regularity to enhance unmixing performance.

To overcome the aforementioned issues, this article introduces the sparsity-enhanced convolutional decomposition (SeCoDe), providing a novel tensor-based solution for solving the blind HU task. To be specific, our contributions can be summarized as follows.

1) We propose a novel SeCoDe decomposition strategy for tensor-based blind HU, in which a two-layered sparsity regularization is designed to further exploit the structural information of abundance maps underlying HSI.

2) To the best of our knowledge, this is the first time to investigate the blind HU issue in a convolutional representation manner. Different from the traditional sparse unmixing models that focus HU at the pixel level, SeCoDe can progressively decompose the HSI from local attention to global aggregation, which enables the full utilization of spatial contextual information by a convolution operation.

3) Beyond the classic bandwise convolutional sparse coding (CSC), a multichannel version (MC-CSC) is developed in our model to learn a continuous spectral representation, yielding a spectrally meaningful component decomposition. The endmember products obtained by the MC-CSC can be explained by spectral bundles, which is robustly against various SVs to a great extent.

4) Accordingly, we provide an effective solution based on an alternative optimization strategy (AOS) with alternating direction method of multipliers (ADMM) for the proposed SeCoDe. Extensive experiments implemented on a series of synthetic and real-world hyperspectral data sets demonstrate the superiority of the proposed method, both visually and quantitatively.

The rest of this article is organized as follows. We first introduce some necessary notions and preliminaries in Section II. Then, representative related works are investigated in Section III. Section IV presents our methodology for the blind HU task. Extensive experimental results are reported and discussed in Section V. Finally, we give concluding remarks and future prospects in Section VI.

II. NOTATIONS AND PRELIMINARIES

The notations in this article are standard in signal processing. Throughout this article, scalars, vectors, and matrices are denoted as the nonbold case, bold lower case, and bold upper case letters, respectively. In addition, the symbol | denotes the outer product of two vectors, i.e., $p \odot q = pq^T$; $\circ$ means Schur–Hadamard products; $\mathbb{R}$ marks the real coordinate space with specific dimensions; $\geq$ represents elementwise inequality; $1$ is all-one vector/matrix with appropriate size; $\odot$ represents variable in the discrete Fourier transform (DFT) domain; and $\| \cdot \|_1$, $\| \cdot \|_F$, and $\| \cdot \|_2$ are the matrix $\ell_1$-norm, the Frobenius norm, and the vector $\ell_2$-norm, respectively.

We denote an $N$-order tensor as $\mathcal{X} = (x_{i_1,...,i_N}) \in \mathbb{R}^{i_1 \times i_2 \times \cdots \times i_N}$, where $i_n = 1, 2, \ldots, I_n$. The mode-$n$ vectors of $\mathcal{X}$ are a series of $I_n$ dimensional vectors extracted from $\mathcal{X}$ by varying other indices. The mode-$n$ unfolding matrix $X_{(n)} \in \mathbb{R}^{i_n \times (i_1 \cdot \cdots \cdot i_{n-1} \cdot i_{n+1} \cdot \cdots \cdot i_K)}$ is obtained by stacking the mode-$n$ vectors as columns. The mode-$n$ product between $\mathcal{X}$ and a matrix $P \in \mathbb{R}^{J \times i_n}$ is denoted by $\mathcal{X} \times_n P$, resulting in an $N$-order tensor $Z \in \mathbb{R}^{i_1 \times \cdots \times i_N}$ with its elements $z_{i_1,...,j,...,i_N} = \sum_{i_n} x_{i_1,...,i_{n-1},j,i_{n+1},...,i_K} P_{ijn}$. A tensor is called rank-1 if it can be written as the outer product of $N$ vectors as $\mathcal{X} = m_{i_1} \odot m_{i_2} \odot \cdots \odot m_{i_N}$, which is also called Kronecker basis. The $\ell_1$-norm and the Frobenius norm of $\mathcal{X}$ are defined as $\| \mathcal{X} \|_1 = \sum_{i_1,...,i_N} |x_{i_1,...,i_N}|$ and $\| \mathcal{X} \|_F = \left( \sum_{i_1,...,i_N} |x_{i_1,...,i_N}|^2 \right)^{1/2}$, respectively.

III. RELATED WORK

In this section, we first briefly introduce the background of the widely studied LMM. Next, we review several representative variants of LMM against SVs and existed tensor factorization-based methods for blind HU, respectively.

A. Conventional LMM

Let $Y = [y_1, y_2, \ldots, y_N]^T \in \mathbb{R}^{N \times L}$ be an $L$-spectrum measured HSI with $N = H \times W$ pixels, $A = [a_1, a_2, \ldots, a_K]^T \in \mathbb{R}^{K \times L}$ be the mixing matrix of $K$ endmembers, and the abundance maps $S = [s_1, s_2, \ldots, s_N]^T \in \mathbb{R}^{N \times K}$ associated with each endmember. The LMM assumes that, for the $n$th pixel, the spectral responses of distinct endmembers bear no interference with each other, which can be written as

$$y_n = s_n^T A + e_n$$  \hspace{1cm} (1)
where $e_n \in \mathbb{R}^{1 \times L}$ represents the additive noise. To account for the physical conditions in reality, three reasonable constraints are commonly imposed, i.e., the sum-to-one constraint of $s_n$ (ASC), the nonnegativity constraint of $s_n$ (ANC), and $a_k$. By collecting all pixels, we can rewrite the LMM in a compact matrix form as

$$Y = SA + E, \quad \text{s.t. } S, A \succeq 0, S_k = I_N. \quad (2)$$

As only $Y$ is available in real scenarios, the above LMM is equivalent to the linear blind source separation (BSS) model in [34]. Given the number of endmembers $K$, a common solution is to first apply the VCA for extracting endmember signatures, and then, fully/partially constrained least-squares unmixing (FCLSU/PCLSU) [35], [36] and sparse unmixing by variable splitting and augmented Lagrangian (SU)L(SAL) [37] can be adopted to estimate the corresponding abundance maps.

### B. LMM Addressing Spectral Variability

The main drawback of conventional LMM lies in its failure to explain the issue of SVs, which potentially hampers an accurate unmixing. For this reason, many LMM-based variants have been proposed for modeling SVs in the process of unmixing. Intuitively, the LMM and its variants can be well generalized into a unified framework as follows:

$$Y = f(S, A) + E \quad (3)$$

where $f$ is a linear function with respect to $S$ and $A$. According to the assumption that SVs are dominated by scaling factors [18], scaled PCLSU (SPCLSU) [38] and scaled SUUnSAL (SUUnSAL) [39] first introduced a diagonal matrix $P_1 \in \mathbb{R}^{N \times N}$ containing shared scaling factors across endmembers, resulting in $f(S, A) = P_1SA$, to allow a pixelwise variation. Instead, extended LMM (ELMM) [40] suggested to adjust endmembers by multiplying a scaling factor to locally absorb contrast and illumination effects, i.e., $f(S, A) = P_2 \circ SA$, where $P_2 \in \mathbb{R}^{N \times K}$.

To overcome other kinds of SVs, Thouvenin et al. [41] proposed a perturbed LMM (PLMM) by adding a perturbation term for each endmember. In this case, $f(S, A) = SA + [s_1^T \Delta_1; s_2^T \Delta_2; \ldots; s_K^T \Delta_N]$, where the rows in $\Delta_n \in \mathbb{R}^{K \times L_n}$ are composed of perturbations of each endmember on the $n$th pixel. Furthermore, Hong et al. [31] extended ELM to an augmented LMM (ALMM) by considering $f(S, A) = P_1SA + S'A'$, where $S'$ and $A'$ are the SV dictionary and corresponding coefficients, respectively. An alternative is the subspace unmixing with low-rank attribute embedding (SULoRA) [30], as formulated by $f(S, A) = SAQ$, where $Q \in \mathbb{R}^{L \times L}$ is the low-rank subspace projection.

### C. LMM Considering Tensor Factorization

Although enormous efforts have been made by the above-mentioned LMMs to address SVs in the unmixing process, these models inevitably suffer from the loss of structural information caused by their improper 2-D formulations. Recently, some tentative methods have been proposed to decompose the HSI from a tensor perspective in order to provide a more compact representation of hyperspectral data in both spatial and spectral domains. More specifically, instead of unfolding 3-D HSI into an observed matrix, tensor-based approaches perform unmixing by means of the tensor factorization technique directly acting on the original 3-D hyperspectral cube.

Zhang et al. [32] first introduced canonical polyadic decomposition (CPD) and applied it in identifying materials of a space object utilizing HSIs. Given a third-order tensor $X' \in \mathbb{R}^{I_1 \times I_2 \times I_3}$, CPD seeks to find a linear combination of $K$ rank-1 tensors

$$X' = \sum_{k=1}^{K} c_k u_k \otimes v_k \otimes w_k \quad (4)$$

where $u_n \in \mathbb{R}^{l_n \times 1}$ denotes the factor vectors of so-called Kronecker basis $u_k$ and imposed coefficient $c_k$ [42].

By following this, Qian et al. [33] designed the matrix-vector nonnegative tensor factorization (MVNTF) for HU by linking the LMM with tensor notation, which was further improved by pursuing local smoothness and nonlocal low-rankness priors [43], [44]. MVNTF can be regarded as
a special case of block term decomposition (BTD) [45]. The BTD can be written as follows:

$$\mathcal{X} = \sum_{k=1}^{K} \mathcal{R}_k \times_1 \mathbf{V}_k^1 \times_2 \mathbf{V}_k^2 \times_3 \mathbf{V}_k^3$$

where \( \mathcal{R}_k \in \mathbb{R}^{R_1 \times R_2 \times R_3} \) is defined as the core tensor, and \( \mathbf{V}_k^1 \in \mathbb{R}^{R_n \times 1}, 1 \leq k \leq K, 1 \leq n \leq 3 \) gathers \( R_n \) orthogonal bases along the \( n \)th mode of \( \mathcal{X} \) (or called terms). By appropriately setting \( R_1 = R_2 = L, R_3 = 1, \mathcal{R}_k = \mathbf{I} \in \mathbb{R}^{L \times L \times 1} \) to be an identity matrix, and \( \mathbf{v}_k^3 \in \mathbb{R}^{1 \times 1} (1 \leq k \leq K) \), we then have the formulation of MVNTF

$$\mathcal{X} = \sum_{k=1}^{K} \mathbf{W}_k \circ \mathbf{v}_k^3$$

where \( \mathbf{W}_k \in \mathbb{R}^{L \times L} \) denotes the product of first two factor matrices. It should be noted that both CPD and MVNTF can be concluded into the unified BTD framework with different ranks as rank-(1, 1, 1) and rank-(\( L, L, 1 \)) terms, respectively.

IV. PROPOSED SecoDe MODEL AND SOLVING ALGORITHM

A. Method Overview

To effectively address SVs and simultaneously preserve spatial–spectral structure information in HU, we seek to directly decompose the 3-D hyperspectral tensor into a series of convolutional kernels and corresponding feature maps in a deconvolutional manner\(^1\) [see the example in Fig. 1(d)], where these learned 3-D convolutions can be well explained by spectral bundles and feature maps can be accordingly seen as abundance maps. Moreover, the 3-D convolutional kernels can be further decomposed into to-be-extracted pure endmembers that preserve the approximately continuous spectral proprieties and 2-D convolutional kernels that are capable of effectively modeling spatial relation robustly against SVs. In addition, our SecoDe model is further improved in unmixing performance by adaptively updating the convolutional dictionary and designing a two-layered sparsity-enhanced regularization.

B. Model Formulation

Before stepping into the proposed SecoDe, we start with a brief introduction of MVNTF because the MVNTF model is not only the cornerstone of our method but also holding clearly mathematical interpretation, as well as easy implementation. Given the observed HSI \( \mathcal{Y} \in \mathbb{R}^{H \times W \times L} \) and its abundances collected in tensor form \( \mathbf{S} \in \mathbb{R}^{H \times W \times K} \), we can rewrite the degradation model from (2) under MVNTF framework as

$$\mathcal{Y} = \sum_{k=1}^{K} \mathbf{S}(k) \circ \mathbf{a}_k + \mathcal{E}, \quad \text{s.t. } \mathbf{S}, \mathbf{A} \succeq 0, \sum_{k=1}^{K} \mathbf{S}(k) = \mathbf{I} \quad (7)$$

where \( \mathbf{S}(k) \in \mathbb{R}^{H \times W} \), i.e., the frontal slice of \( \mathbf{S} \), denotes the abundance map corresponding to the \( k \)th endmember. A straightforward solution is turning (7) into a matrix-based minimization problem of mean square error by mode-3 unfolding. Alternatively, an alternating least-squares (ALS) algorithm was given in [33] by iteratively solving matrix factorization problems to each mode unfolded matrix of \( \mathcal{Y} \). However, the factors along spatial modes with enforced low-rankness and nonnegativity properties still lack intuitive interpretations.

1Unlike existing deconvolution-based unmixing methods that mostly only use a single filter to characterize the blur degradation [46], we here propose to adaptively learn overcomplete convolutional kernels to capture the more detailed spatial structure and directly represent the HSIs.
Therefore, the well-designed MC-CSC has been applied to model spectral information in our SeCoDe continuously, i.e.,

\[ S = \sum_{d=1}^{D} F_d \otimes M_d \]  

(8)

where \( D \) is the size of filter bank \( \mathcal{F} = \{ F_d \}_{d=1}^{D} \), and the third-order tensor \( M_d \in \mathbb{R}^{H \times W \times K} \) is constructed by stacking \( K \) feature maps \( \{ M_d^{(k)} \}_{k=1}^{K} \) corresponding to the \( d \)th filter \( F_d \in \mathbb{R}^{P \times P} \) along the third dimension. By plugging (8) into (7), the 3-D HSI \( Y \) can be reconstructed in a tensor-based fashion, as illustrated in Fig. 1(d).

Note that, for the abundance map of each endmember, we have \( S^{(k)} = \sum_{d=1}^{D} F_d \otimes M_d^{(k)} \). Let the linear operator \( M_d^{(k)} \in \mathbb{R}^{H \times W \times P^2} \) satisfy \( M_d^{(k)} \vec{v}(F_d) = \vec{v}(F_d \otimes M_d^{(k)}) \), where \( \vec{v}(\cdot) \) denotes the common vectorization. Therefore, the mode-3 unfolding matrix of the hyperspectral cube \( Y \), i.e., \( Y_{(3)} \in \mathbb{R}^{L \times N} \), can be approximated by

\[ Y_{(3)}^T \approx \sum_{k=1}^{K} \left( \sum_{d=1}^{D} M_d^{(k)} \vec{v}(F_d) \right) a_k^T \]

\[ = \sum_{k=1}^{K} \sum_{d=1}^{D} M_d^{(k)} B_d^{(k)} \]  

(9)

where \( B \) represents 3-D convolutional kernels that consist of \( B_d^{(k)} = \vec{v}(F_d) a_k^T \in \mathbb{R}^{P^2 \times L} \), \( k = 1, \ldots, K \), and \( P^2 \) stands for the number of pixels contained in each filter \( F_d \). With this form of representation, \( B_d^{(k)} \) and \( \bar{M}_d^{(k)} \) clearly exhibit same physical meaning as spectral bundles and corresponding fractional maps. The complex SVs are, therefore, expected to be absorbed by such a linear combination. Furthermore, we have the following tensor-based decomposition model with the given filter bank \( \mathcal{F} \):

\[ \min_{\Theta} \frac{1}{2} \| Y_{(3)} - \sum_{k=1}^{K} \left( \sum_{d=1}^{D} M_d^{(k)} \vec{v}(F_d) \right) a_k^T \|^2. \]  

(10)

To integrate spectral bundle into unique endmember for each material and reduce the model complexity, we relax the model in (10) into two terms by

\[ \min_{\Theta} \frac{1}{2} \left\| Y - \sum_{k=1}^{K} S^{(k)} \circ a_k \right\|^2_F + \frac{\alpha}{2} \left\| S - \sum_{d=1}^{D} F_d \otimes M_d \right\|^2_F \]  

(11)

where \( \Theta = \{ A, M, S, \mathcal{F} \} \) denotes the collection of to-be-estimated variables. With commonly used priors, e.g., sparsity-promoting term in the form of \( \ell_1 \)-norm, and necessary constraints, our baseline in (11) can be further written as

\[ \min_{\Theta} \frac{1}{2} \left\| Y - \sum_{k=1}^{K} S^{(k)} \circ a_k \right\|^2_F + \frac{\alpha}{2} \left\| S - \sum_{d=1}^{D} F_d \otimes M_d \right\|^2_F \]  

\[ + \beta \sum_{d=1}^{D} \| M_d \|_1 \]

s.t. \( S, A \succeq 0, \sum_{k=1}^{K} S^{(k)} = 1 \)  

(12)

where \( \alpha \) and \( \beta \) are positive weighting parameters.

C. Performance Amelioration

Since the sparse coding problems for distinct abundance maps are decoupled, we readily make the following extensions to further embrace the amelioration of model performance.

1) Dictionary Update: To enhance the generalization ability of convolutional operations in SeCoDe, we adaptively update the convolutional filters (or dictionary), namely, \( \mathcal{F} \), according to the currently investigated hyperspectral scene. Fig. 3 visualizes some examples of convolutional filters with various learned patterns.

2) Two-Layered Sparsity Regularization: A reasonable assumption in the case of HU is that the observed spectrum in each pixel is probably contributed by a small set of endmembers [37]. Motivated by the recent success of sparsity-promoting enhancement in different domains [48], [49], we propose to double sparsity in the form of a two-layered sparsity regularization to further highlight the structural information in to-be-estimated maps and reduce the redundancy simultaneously. Note that other sparsity-promoting regularization terms with the forms of \( \ell_{1/2} \) quasi-norm [50] and nonconvex log-sum function [23] are also optional.

Let \( \Theta' = \{ A, M, S, \mathcal{F} \} \) collect all the unknown parameters, and the proposed SeCoDe aims to solve

\[ \min_{\Theta'} \frac{1}{2} \left\| Y - \sum_{k=1}^{K} S^{(k)} \circ a_k \right\|^2_F + \frac{\alpha}{2} \left\| S - \sum_{d=1}^{D} F_d \otimes M_d \right\|^2_F \]  

\[ + \beta \sum_{d=1}^{D} \| M_d \|_1 + \gamma \sum_{k=1}^{K} \| S^{(k)} \|_1 \]

s.t. \( \Theta' = \Omega \)  

(13)

where

\[ \Omega = \{ M_d^{(k)} \geq 0, a_k \geq 0, S^{(k)} \geq 0 \ \forall d, k \} \]

\[ \sum_{k=1}^{K} S^{(k)} = 1 \]

\[ \| F_d \|_F^2 = 1. \]

D. Model Optimization

The objective function in (13) is a typical nonconvex problem with respect to the unknown variable set \( \Theta' \); thus, it is difficult to directly obtain the analytical solution in one step. A common solution for our case is to divide the original optimization problem into two easily solved
subproblems relying on an AOS that alternatively optimizes each subproblem when the other is fixed, while each subproblem can be efficiently deduced via the ADMM solver. More specifically, the two subproblems can be unfolded in the following. A comprehensive implementation for SeCoDe is given in Algorithm 1.

1) MC-CSC: The first subproblem can be featured as a special CSC with multichannel attentions, jointly updating convolutional filters and multichannel maps, i.e.,

$$
\min_{\mathcal{F}, \mathcal{M}} \frac{\alpha}{2} \|\mathcal{S} - \sum_{d=1}^{D} \mathbf{F}_d \otimes \mathcal{M}_d\|_F^2 + \beta \sum_{d=1}^{D} \|\mathcal{M}_d\|_1
$$

s.t. \( \mathcal{M}_d \succeq 0, \quad \|\mathbf{F}_d\|_F^2 = 1 \quad \forall d \) (15)

which can be efficiently solved using ADMM in an inexact fashion \cite{47,51-53}.

With the filters \( \mathcal{F} \) fixed, we first update the sparse coefficient map bank \( \mathcal{M} = \{\mathcal{M}_d\}_{d=1}^{D} \) by solving a set of \( K \) independent subproblems in parallel

$$
\min_{\mathbf{M}_d^{(k)} \succeq 0} \frac{1}{2} \|\mathbf{S}^{(k)} - \sum_{d=1}^{D} \mathbf{F}_d \otimes \mathbf{M}_d^{(k)}\|_F^2 + \frac{\alpha}{2} \sum_{d=1}^{D} \|\mathbf{M}_d^{(k)}\|_{1,1} \quad \forall k. 
$$

(16)

For each \( k \)th subproblem, we omit the superscript for simplicity, by defining

$$
\tilde{\mathbf{F}} = [\tilde{\mathbf{F}}_1, \ldots, \tilde{\mathbf{F}}_D], \quad \mathbf{M} = [\mathbf{M}_1^T, \ldots, \mathbf{M}_D^T]^T
$$

(17)

where \( \tilde{\mathbf{F}}_d \) is a linear operator such that \( \tilde{\mathbf{F}}_d \mathbf{M}_d = \mathbf{F}_d \otimes \mathbf{M}_d \); we can rewrite each (16) into a compact form

$$
\min_{\mathbf{M} \succeq 0} \frac{1}{2} \|\mathbf{S} - \tilde{\mathbf{F}} \mathbf{M}\|_F^2 + \frac{\alpha}{\beta} \|\mathbf{M}\|_{1,1}. 
$$

(18)

We introduce auxiliary variable \( \mathbf{O} = \mathbf{M} \), obtaining the corresponding augmented Lagrangian function in the scaled version

$$
\mathcal{L}(\mathbf{M}, \mathbf{O}, \mathbf{\Gamma}) = \frac{1}{2} \|\mathbf{S} - \tilde{\mathbf{F}} \mathbf{M}\|_F^2 + \frac{\nu_1}{2} \|\mathbf{M} - \mathbf{O} + \mathbf{\Gamma}_1\|_F^2 + \frac{\alpha}{\beta} \|\mathbf{O}\|_{1,1} + l^+_h(\mathbf{O})
$$

(19)

where \( \mathbf{\Gamma}_1 \) is the Lagrange multiplier, \( \nu_1 \) is a positive penalty parameter, and \( l^+_h(\cdot) \) acts as a positive-enforcement operator. Then, under the ADMM framework, we can solve the problem by iteratively updating the three variables.

For the variable \( \mathbf{M} \), Wohlberg \cite{47} proposed to exploit the fast Fourier transform (FFT) in equivalently solving independent linear systems

$$
(\tilde{\mathbf{F}}^H \tilde{\mathbf{F}} + \nu_1 \mathbf{I}) \mathbf{M} = (\tilde{\mathbf{F}}^H \mathbf{S} + \nu_1 (\hat{\mathbf{O}} - \hat{\mathbf{\Gamma}}_1))
$$

(20)

where \( (\cdot)^H \) denotes the conjugate transpose of a complex matrix. The solution can be efficiently computed by applying the Sherman-Morrison formula.

The updates of \( \mathbf{O} \) and \( \mathbf{\Gamma} \) are readily solved by

$$
\mathbf{O} = \max(0, D_{\nu_1/\nu_1}(\mathbf{M} + \mathbf{\Gamma}_1)) \quad \mathbf{\Gamma}_1 = \mathbf{\Gamma}_1 + \mathbf{M} - \mathbf{O}
$$

(21-22)

where \( D_{\nu}(\cdot) \) is the soft thresholding operator, i.e., \( D_{\nu}(x) = \text{sign}(x) \max(0, |x| - \nu) \).

After updating the sparse coefficient maps, we then fix maps \( \mathcal{M} \) and update filters \( \mathcal{F} \) by solving the following subproblem:

$$
\min_{\mathbf{F}_d} \frac{1}{2} \sum_{k=1}^{K} \left\|\mathbf{S}^{(k)} - \sum_{d=1}^{D} \mathbf{F}_d \otimes \mathbf{M}_d^{(k)}\|_F^2, \quad \text{s.t.} \quad \|\mathbf{F}_d\|_F^2 = 1 \quad \forall d.
$$

(23)

Li et al. \cite{54} utilized a proximal gradient descent method to solve it. Instead, we adopt ADMM again, for its prospect of interleaving the ADMM iterations for two subproblems as a whole, which ensures high efficiency and promising practicability \cite{47}.

We can rewrite (23) by vectorizing variables and defining \( \mathbf{P} \) as zero-padding operator

$$
\min_{\mathbf{F}_d} \frac{1}{2} \sum_{k=1}^{K} \left\|\mathbf{s}^{(k)} - \sum_{d=1}^{D} \mathbf{f}_d \otimes \mathbf{m}_d^{(k)}\|_2^2, \quad \text{s.t.} \quad \mathbf{f}_d \in C_{P_0} \quad \forall d.
$$

(24)
where \( C_{pn} = \{ f \in \mathbb{R}^N | (I - PP^T)f = 0, \|f\|_2 = 1 \} \) is the constraint set. Similarly, we can rewrite its scaled Lagrangian function with auxiliary variable \( t_d = f_d^* \):

\[
\mathcal{L}(f_d, t_d) = \frac{1}{2} \sum_{k=1}^{K} \left\| S(k) - \sum_{d=1}^{D} f_d \odot \mathbf{m}_d(k) \right\|_2^2 + \sum_{d=1}^{D} \delta_{C_{pn}}(t_d) + \frac{\nu_2}{2} \sum_{d=1}^{D} \| f_d - t_d + \Gamma_d \|_2^2
\]

(25)

where \( \delta_{C_{pn}} \) is the indicator function that penalizes the out-of-set solution.

The padded filter \( f_d \) is updated in the DFT domain by

\[
\min_F \frac{1}{2} \sum_{k=1}^{K} \| S(k) - \tilde{M}(k)^T \tilde{F} \|_2^2 + \frac{\nu_2}{2} \| \tilde{F} - \tilde{T} + \tilde{\Gamma}_2 \|_2^2
\]

(26)

where \( \tilde{M}(k) = [M_1(k), \ldots, M_M(k)] \) with \( \tilde{M}_d(k) = \text{diag}(\mathbf{m}_d(k)) \in \mathbb{R}^{N \times N} \), and \( \tilde{F} = [f_1^T, \ldots, f_D^T]^T \), similar for \( \tilde{T} \) and \( \tilde{\Gamma}_2 \). By following (20), we can get the solution by solving:

\[
\left( \sum_{k=1}^{K} (\tilde{M}(k))^T \tilde{M}(k) + \nu_2 I \right) \hat{F} = \left( \sum_{k=1}^{K} (\tilde{M}(k))^T \tilde{S}(k) + \nu_2 (\tilde{T} - \tilde{\Gamma}_2) \right)
\]

(27)

with the iterated Sherman–Morrison formula.

The auxiliary variable \( t_d \) and dual variables \( \Gamma_d \) are updated in forms of

\[
t_d = \frac{PP^T (f_d + \Gamma_d)}{\|PP^T (f_d + \Gamma_d)\|_2^2}
\]

(28)

\[
\Gamma_d = \Gamma_d + t_d - f_d.
\]

(29)

More details in solving the MC-CSC are then summarized in Algorithm 2.

2) Coupled Spectral Unmixing (CSU): Once the convolutional kernels and sparse feature maps are obtained by solving the MC-CSC, the second part can further integrates them into a set of unique endmembers and corresponding abundance maps by solving the following CSU:

\[
\min_{S, A} \frac{1}{2} \left\| \gamma' - \sum_{k=1}^{K} S(k) \circ a_k \right\|_F^2 + \frac{\alpha}{2} \| S - Z \|_F^2 + \gamma \| S \|_1
\]

s.t. \( S, A \geq 0, \sum_{k=1}^{K} S(k) = 1 \)

(30)

where \( Z = \sum_{d=1}^{D} F_d \otimes M_d \).

We first introduce multiple auxiliary variables \( G, H, \) and \( J \), obtaining the scaled augmented Lagrangian function as follows:

\[
\mathcal{L} = \frac{1}{2} \left\| \gamma' - S^T(3) A - Z^T(3) \right\|_F^2 + \frac{\alpha}{2} \left\| S^T(3) - Z^T(3) \right\|_F^2 + \gamma \| J \|_{1,1}
\]

\[
+ \frac{\mu}{2} \left\| S^T(3) - H + \lambda_2 \right\|_F^2 + \frac{\mu}{2} \left\| S^T(3) - J + \lambda_3 \right\|_F^2
\]

\[
+ \frac{\mu}{2} \left\| A - G + \lambda_1 \right\|_F^2 + \frac{\mu}{2} \left\| G + \lambda_1 \right\|_F^2 + \frac{\mu}{2} \left\| \lambda_1 \right\|_F^2
\]

(31)

where \( \{ \lambda_i \}_{i=1}^3 \) are the Lagrange multipliers, and \( \mu \) is the positive penalty parameter. By setting the derivatives of (31) with respect to each unknown variable to zero, we can readily obtain the updating rules as

\[
\left\{ \begin{array}{l}
S^T(3) = \left[ (Y^T(3) A^T + \alpha z^T(3) + \mu (H - J + \lambda_2 - \lambda_3)) / (AA^T + (\alpha + 2\mu)I) \right] \\
A = (S^t(3) S^t(3) + \mu I) \backslash S^t(3) Y^T(3) + \mu (G - \lambda_1) \\
G = \text{max}(0, A + \lambda_1), \quad H = \text{max}(0, S^T(3) + \lambda_2) \\
J = D_{\lambda_1} / \mu \left( S^T(3) + \lambda_1 \right) \\
\lambda_1 = \lambda_1 + A - G, \quad \lambda_2 = \lambda_2 + S^T(3) - H \quad \lambda_3 = \lambda_3 + S^T(3) - J
\end{array} \right.
\]

(32)

(33)

(34)

(35)

where \( \backslash \) denotes matrix left division. Algorithm 3 details the specific steps for solving the CSU’s subproblem.

Finally, we repeat these optimization procedures until a presetting stopping criterion is satisfied.

3) Computational Complexity: The computational cost of the proposed SeCoDe is mainly dominated by the above two subproblems. The ADMM solver of the latter CSU only involves matrix multiplication, addition, and inversion, yielding \( O(KLN) \). As for the former MC-CSC subproblem, it is the cost of the FFTs and linear systems solvers for (20) and (27) that dominate the computational complexity, whereas the cost can be generally rated as \( O(DN \log N) \) by virtue of applying the (iterated) Sherman–Morrison formula.

V. EXPERIMENTS AND DISCUSSION

In this section, we conduct elaborated experiments on one synthetic and two real data sets, as shown in Fig. 4, to exhibit the effectiveness of proposed SeCoDe for blind HU. We compare the proposed method with seven representative and state-of-the-art unmixing methods, including FCLSU [35], PCLSU [36], SCPLSU [38], SUSUnSAL [39], ELMM [40],
MVNTF [33], SULoRA [30], and ALMM [31]. Some of these methods, such as SULoRA and ALMM, are extended to a blind unmixing version, making them able to simultaneously estimate abundance maps and extract endmembers.

A. General Experimental Setup

For a fair comparison, we optimally tune the parameters involved in competitive methods and report the average results out of ten runs. All experiments are implemented with MATLAB 2017b on a Windows 10 Operation System and conducted on Intel Core i7-8700K 3.70-GHz desktop with 64-GB memory.

1) Implementation Details: Since the details of implementing CSC are not the focus of this article, for space brevity, we leave out the discussion on the number of filters $D$, the filter size $p$, and different initialization. We empirically set them as 36 and 12, after weighing the performance gain and extra computational cost. The filters are simply randomly initialized with the central 1/9 part. In addition, several important regularization parameters ($\alpha, \beta,$ and $\gamma$) in our SeCoDe will be emphatically discussed and analyzed in Section V-B2.

In our case, the number of endmembers $K$ can be determined by using hyperspectral signal identification by minimum error (HySime) [55], and the endmembers are further extracted by VCA. Due to the highly ill-posed problem and strong nonconvexity of the model, a bad initialization may render the algorithm converge into a local minimum. Hence, we select the output (abundance maps) of SPCLSU as a good starting point. Finally, the materials can be identified by similarity measurement (spectral angle) between references and estimated endmembers.

2) Performance Metrics: We introduce three performance metrics to quantitatively assess the experimental results. Given faithful references $\mathbf{S}$ and $\mathbf{A}$, the abundance overall root mean square error (aRMSE)

$$\text{aRMSE} = \frac{1}{N} \sum_{n=1}^{N} \sqrt{\frac{1}{K} \| s_n - \hat{s}_n \|_2^2}$$

and the spectral angle distance (SAD)

$$\text{SAD} = \frac{1}{K} \sum_{k=1}^{K} \arccos \left( \frac{\mathbf{a}_k^T \hat{\mathbf{a}}_k}{\| \mathbf{a}_k \|_2 \| \hat{\mathbf{a}}_k \|_2} \right)$$

are utilized to assess the fidelity of estimated abundance maps $\hat{\mathbf{S}}$ and endmembers $\hat{\mathbf{A}}$, respectively.

Due to the lack of fully reliable ground truth for abundances in the real scene, we additionally perform classification in terms of the overall accuracy (OA), to relax the requirement for references with high fidelity and give a more comprehensive assessment. The classification maps can be obtained by identifying each pixel into a certain endmember that has the maximal response.

B. Synthetic Data

1) Data Description: The synthetic data have been widely applied in [30], [31], and [40] to evaluate the unmixing performance quantitatively. It consists of $200 \times 200$ pixels with 224 spectral bands in the wavelength ranging from 400 to 2500 nm. Moreover, five materials selected from USGS spectral library are investigated in the studied scene. Note that the image is simulated by adding scaling factors and complex noises in order to assess the unmixing ability in the presence of SV. Please refer to [40] for more details regarding this data.

2) Parameter Sensitivity Analysis: Since the unmixing performance is sensitive to the setting of three regularization parameters $\alpha, \beta,$ and $\gamma$ in SeCoDe to some extent, it is, therefore, indispensable to find a proper range to guide the determination of parameters. After that, we adopt a grid search strategy throughout our experiments. To be specific, we iteratively evaluate the optimal value of each parameter while fixing the others, until an optimal setting with the lowest performance metrics is reached. As shown in Fig. 5, the optimal values with respect to the three parameters, i.e., $(2\epsilon - 2, 1 \epsilon - 2, 3)$, can be easily found with approximately convex curves.

3) Parameter Setting: The parameters for all the algorithms are recorded as follows. For the SunSAL, the sparsity-promoting regularization is weighed by $2\epsilon - 3$. We set the regularization parameter in ELMM as $5\epsilon - 1$. The parameter combination for SULoRA is selected as $(1\epsilon - 2, 1 \epsilon - 2, 4 \epsilon - 4)$, and ALMMs’ is $(4 \epsilon - 3, 2 \epsilon - 3, 5 \epsilon - 3)$.

4) Results and Discussion: Table I summarizes the aRMSEs and SADs for all competitive methods. We put the abundance maps of ground truth in the left-hand column of Fig. 6. To highlight the differences of estimated abundance maps for different methods, we show the residual images between the abundance maps and GT in Fig. 6.

As listed in Table I, we can conclude that SeCoDe achieves the best results in terms of both metrics. The visual results presented in Fig. 6 exhibit a consistent tendency. Specifically, due to strictly following ASC, FCLSU inevitably absorbs SVs in abundance estimation. For that, PCLSU relaxes the
TABLE I

<table>
<thead>
<tr>
<th>Metric</th>
<th>Ideal Value</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>aRMSE</td>
<td>0</td>
<td>FCLSU</td>
</tr>
<tr>
<td>SAD</td>
<td>0</td>
<td>PCLSU</td>
</tr>
<tr>
<td></td>
<td>0.0168</td>
<td>SPCLSU</td>
</tr>
<tr>
<td></td>
<td>0.0033</td>
<td>SSUnSAL</td>
</tr>
<tr>
<td></td>
<td>0.0356</td>
<td>ELMM</td>
</tr>
<tr>
<td></td>
<td>0.0424</td>
<td>MVNTF</td>
</tr>
<tr>
<td></td>
<td>0.0232</td>
<td>SULoRA</td>
</tr>
<tr>
<td></td>
<td>0.0226</td>
<td>ALMM</td>
</tr>
<tr>
<td></td>
<td>0.0202</td>
<td>SeCoDe</td>
</tr>
<tr>
<td>Runtime (Sec.)</td>
<td>-</td>
<td>2.82</td>
</tr>
<tr>
<td></td>
<td>1.62</td>
<td>1.67</td>
</tr>
<tr>
<td></td>
<td>2.10</td>
<td>434.61</td>
</tr>
<tr>
<td></td>
<td>545.34</td>
<td>18.31</td>
</tr>
<tr>
<td></td>
<td>184.91</td>
<td>223.19</td>
</tr>
</tbody>
</table>

ASC in FCLSU and, hence, alleviates absorption of SVs to some extent, which brings an evident improvement. Scaled methods, such as SSUnSAL and SPCLSU, further increase the performance by modeling the main constituent of SVs, i.e., scaling factors. The results of MVNTF show evident rough patterns; the possible reason is the lack of consideration of strong SVs contained in this scene. Although the ELMM is designed with meaningful physical consideration, the complex optimization makes the model easily trapped in local minima. As a result, it performs an unsatisfactory estimation of abundances, as also indicated by the salient region with dark colors on the residual map of material 2. Extended from ELMM, ALMM successfully avoids such a dilemma and produces the second best result. The result of SULoRA comes in third due to its robust low-rank attribute embedding. Different from the dense noises in the results of traditional methods, the SeCoDe’s results are clearer. This means that the errors are mainly located in fewer pixels, which declares a more faithful result as a whole.

C. Real Data—Samson

1) Data Description: The Samson data set was captured by a push broom, visible to the near-infrared sensor and well-calibrated. The original image contains 952 × 952 pixels measured in 156 bands ranging from 401 to 889 nm, with a spectral resolution highly up to 3.13 nm. We choose a region of interest (ROI) with the size of 95 x 95 pixels. The referential abundance maps of three appeared endmembers, i.e., “#1 Rock,” “#2 Tree,” and “#3 Water,” provided from the website, are shown in Fig. 7 (left).

2) Parameter Setting: We record the tuned parameters for all the algorithms as follows. The sparsity-induced regularization term in SSUnSAL is weighted by $5 \times 10^{-3}$. The tradeoff parameter for two fidelity terms in ELMM is set as $5 \times 10^{-1}$. For SULoRA and ALMM, the optimal parameter combinations are set as $\{1, 10^{-2}, 2 \times 10^{-2}\}$ and $\{2 \times 10^{-4}, 2 \times 10^{-2}, 4 \times 10^{-3}, 1 \times 10^{-3}, 20\}$, respectively. We choose $\alpha$, $\beta$, and $\gamma$ in SeCoDe as $10^{-1}$, $10^{-2}$, and $5 \times 10^{-1}$, respectively.

3) Results and Analysis: Fig. 7 visualizes the abundance maps of different methods, and the corresponding quantitative results are given in Table II. Overall, FCLSU and PCLSU yield relatively poor results as they fail to model SVs. By considering the principal SV-scaling factors, SPCLSU achieves an obvious improvement in aRMSE. ELMM assumes different scaling factors on endmembers, yielding further performance

*https://rslab.ut.ac.ir/data
TABLE II
QUANTITATIVE PERFORMANCE COMPARISON WITH THE INVESTIGATED METHODS IN TERMS OF aRMSE, SAD, AND OA ON THE SAMSON DATA SET. THE BEST ONE IS SHOWN IN BOLD

<table>
<thead>
<tr>
<th>Metric</th>
<th>Ideal Value</th>
<th>Method</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FCLSU</td>
<td>PCLU</td>
<td>SPCLSU</td>
<td>SSUnSAL</td>
<td>ELMM</td>
<td>MVNTF</td>
<td>SULoRA</td>
</tr>
<tr>
<td>aRMSE</td>
<td>0</td>
<td>0.1633</td>
<td>0.1492</td>
<td>0.1207</td>
<td>0.0779</td>
<td>0.1158</td>
<td>0.1336</td>
<td>0.0689</td>
</tr>
<tr>
<td>SAD</td>
<td>0</td>
<td>0.1267</td>
<td>0.0844</td>
<td>0.0844</td>
<td>0.1054</td>
<td>0.1405</td>
<td>0.0732</td>
<td>0.0701</td>
</tr>
<tr>
<td>OA (%)</td>
<td>100</td>
<td>80.41</td>
<td>89.12</td>
<td>89.14</td>
<td>89.14</td>
<td>85.30</td>
<td>88.68</td>
<td>91.24</td>
</tr>
<tr>
<td>Runtime (Sec.)</td>
<td>-</td>
<td>0.42</td>
<td>0.56</td>
<td>0.57</td>
<td>0.54</td>
<td>168.33</td>
<td>432.65</td>
<td>4.80</td>
</tr>
</tbody>
</table>

Fig. 7. Qualitative comparison of abundance maps estimated by the proposed method and state-of-the-art compared methods on the Samson data set.

D. Real Data—Urban

1) Data Description: The second real data set was captured by the Hyperspectral Digital Image Collection Experiment (HYDICE) sensor in October 1995, covering an urban area at Copperas Cove, TX, USA. The original image contains $307 \times 307$ pixels measured in 210 bands ranging from 400 to 2500 nm, at a 2-m ground sample distance (GSD) and a 10-nm spectral resolution. As a common preprocessing procedure, we manually remove severely degraded bands with the water vapor and atmospheric effects, i.e., bands with number 1–4, 76, 87, 101–111, 136–153, and 198–210. Fig. 4(c) shows the data cube with the remaining 162 bands. The widely used references for endmembers and abundance maps can be found on the website. 5

2) Parameter Setting: The optimal parameters for the competitive methods on this data set are $2e-3$ for SSUnSAL, $5e-1$ for ELMM, $\{1, 1e-2, 6e-4\}$ for SULoRA, and $\{2e-2, 2e-2, 6e-3\}$ for ALMM. The $\alpha$, $\beta$, and $\gamma$ in SeCoDe are parameterized by $6e-1$, $1e-2$, and 1, respectively.

3) Results and Analysis: The visual results are shown in Fig. 8, and the quantitative results are listed in Table III. As can be seen from the table, FCLSU and MVNTF yield relatively poor unmixing results with only around 60% OAs.
The metrics on abundance maps of SPCLSU and SSUnSAL are much better, due to the consideration that their estimated endmembers are more accurate. ALMM is able to extract more reliable endmembers and slightly surpass SeCoDe in terms of SAD. However, great advantages of ARMSE and OA show the effectiveness of the proposed method by integrating convolutional representations, which is able to fully capture spatial information underlying hyperspectral data.

More observations can be found in Fig. 8. Although SeCoDe performs a relatively poor visualization on the material of Roof from a numerical perspective, it can provide a more realistic spatial distribution for the abundances of each material. Taking the long horizontal road in the top half of the scene as an example, both MVNTF and PCLSU fail to capture it, whereas FCLSU and ELMM improperly unmix it as an ingredient of Grass. On the other hand, it is mistakenly classified into Roof by the other four competitive methods, which may share similar endmembers since they are both man-made materials. Another big area of asphalt besides the Roof is also difficult to be recognized, whereas it is accurately estimated only by our SeCoDe. These pieces of evidence objectively validate the superiority of the proposed method compared to not only traditional NMF-based methods but also complex methods that addressing SVs.

E. Endmember Visualization

Finally, we analyze the endmembers estimated by our method and references on the three data sets, as visualized in Fig. 9. In the synthetic data, SeCoDe is capable of extracting high-quality endmembers, yielding a good matching with references, especially for material 2 (M2) and material 4 (M4), while, in the real scenarios, the gap between the estimations and references is relatively obvious. A reasonable explanation might be twofold. On the one hand, the complex imaging environment in real cases tends to generate more challenging SVS than in synthetic ones, which possibly brings negative effects in estimating endmembers. On the other hand, the acquisition and reliability for the references of endmembers remain limited; thus, it is difficult to accurately measure the similarities between these estimated endmembers and so-called ground truth. Despite so, the proposed SeCoDe performs relatively realistic estimations for endmembers, particularly in spectral shapes and the positions of band absorption.

VI. CONCLUSION

Motivated by the recent success of tensor analysis in the remote sensing community, we propose a tensor-based convolutional decomposition model, called SeCoDe, with a sparsity-enhanced constraint for the blind HU tasks. SeCoDe
learns 3-D convolutional kernels and regards them as spectral bundles, thereby effectively eliminating the effects of SVs. Moreover, the resulting MC-CSC is able to not only model the spatial contextual information utilizing convolutional operations but also preserve the spectral continuity. To further ameliorate the unmixing performance, we merge an adaptive filter update and a two-layered sparsity regularization into a general framework. With these well-designed strategies, the proposed method achieves the best unmixing performance among experiments on both synthetic and real data sets compared to previous methods. In future work, we will pose the issue of HU and robustly address various SVs.

REFERENCES


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